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Robust Antiwindup for Manual Flight Control of an Unstable Aircraft

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A result on robust-in-the-large antiwindup compensation obtained for the case of asymptotically stable systems is extended to deal with unstable plants with asymptotically null references. The use of a weakened formulation of the antiwindup problem is crucial in the achievement of robustness to large parameter variations. The proposed design approach results in an antiwindup compensator that works as an add-on for any a priori given, high-performance controller, synthesized without taking saturation or robustness issues into account. The technique is applied to a reduced, short-period dynamic model of a highly unstable fighter aircraft, a case in which the issues of control saturation during aggressive maneuvering and robustness with respect to large parameter variations are of paramount importance. Simulations in low- and high-speed regimes demonstrate that the weakened antiwindup compensator outperforms previous antiwindup techniques in providing robust stability to the closed-loop system while exploiting the available control power in all of the considered maneuvers.

Nomenclature

$\boldsymbol{A}, \boldsymbol{B}$	=	state and control matrices of nominal model
$\boldsymbol{A}_c, \boldsymbol{B}_c$	=	state and control matrices of preassigned controller
$\boldsymbol{C}_c, \boldsymbol{D}_c$	=	output matrices of preassigned controller
$\boldsymbol{C}_1, \boldsymbol{D}_1$	=	performance output matrices of nominal model
$\boldsymbol{C}_2, \boldsymbol{D}_2$	=	measured output matrices of nominal model
$\boldsymbol{E}_c, \boldsymbol{F}_c$	=	matrices for reference signal in preassigned controller
F	=	generic n-input, n-output filter
K_{aw}	=	antiwindup compensator
K_M	=	preassigned controller
M	=	saturation limit
P	=	model of system
P_Ψ	=	perturbed model
P_0	=	nominal model
Q	=	dynamic pressure
q	=	pitch angular velocity
$\boldsymbol{r}$	=	reference signal (pilot input)
$\mathcal{U}$	=	compact convex subset of region in $\mathbb{R}^p$ where $\mathrm{sat}(\boldsymbol{u}) = \boldsymbol{u}$
$\boldsymbol{u}$	=	unsaturated control variables
$\mathcal{V}$	=	safe region
$\mathcal{V}_Q$	=	null controllable region at dynamic pressure Q
$\mathcal{X}$	=	domain of attraction
$\mathcal{X}_0$	=	region of nominal behavior
$\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}$	=	state, measured, and performance output variables of system
$\boldsymbol{x}_{\mathrm{aw}}, \boldsymbol{v}_1, \boldsymbol{v}_2$	=	state and output variables of antiwindup compensator
$\boldsymbol{x}_c, \boldsymbol{y}_c, \boldsymbol{u}_c$	=	state, output, and control variables of preassigned controller
$\boldsymbol{x}_F$	=	filter state
$\boldsymbol{x}_M, \boldsymbol{y}_M, \boldsymbol{u}_M$	=	state, output, and control variables of nominal model
$\boldsymbol{y}_\psi$	=	output of perturbation
Z_α, M_α, M_q, M_δ	=	aircraft model stability derivatives
α	=	angle of attack
$\Gamma_{\mathcal{C}}$	=	projection function on convex set $\mathcal{C}$
$\gamma_{y,u}^{(W)}$	=	$\mathcal{L}_2$ induced gain of system W from $\boldsymbol{u}$ to $\boldsymbol{y}$
δ	=	saturated equilibrator deflection
Σ	=	closed-loop system
Ψ	=	perturbation system
$\boldsymbol{\psi}$	=	state variables of perturbation system

Subscripts

S	=	saturated
S_{aw}	=	saturated with antiwindup compensator
U	=	unsaturated
U_{aw}	=	unsaturated with antiwindup compensator
0	=	nominal

Superscripts

-	=	signal in Σ_U
~	=	signal in $\Sigma_{U_{\mathrm{aw}}}$

Introduction

MODERN high-performance fighter aircraft are characterized by an extended flight envelope to allow the pilot to reach unprecedented maneuvering capabilities at high angles of attack. Such a result can be achieved only exploiting the control power of aerodynamic surfaces as much as possible, and this has serious consequences on the challenges for the control engineer. In fact, during aggressive maneuvering, the aerodynamic controls can reach their saturation limit, or, in other cases, the pilot control demand can change abruptly, and rate saturation can affect the overall performance of the control system. Moreover, the coupling of control (magnitude and rate) saturation with the response of a human pilot can be the root for pilot-induced oscillations that were recognized as the cause of several accidents or critical events. (See, for example, Refs. 1–4 and references therein.)

In many cases, instabilities are avoided using so-called flight envelope protection systems, that is, the pilot input is limited or

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even canceled when dangerous zones of the flight envelope are approached. An example in the publicly available literature is the control system of the F-16 fighter aircraft, where it is possible to observe devices such as the automatic spin prevention system or the antistall system.[5] In both cases, pilot commands to rudder and elevator are progressively faded when the angle of attack exceeds a value of 20 deg, until they are canceled for $\alpha > 25$ deg. The parameters of the flight envelope protection system are usually tuned during flight tests, when the test pilot recognizes a tendency to control loss or a serious degradation of handling qualities (HQ).

Similar approaches, based on a variety of mathematical tools, have been proposed for manual flight control in several papers, accounting for magnitude saturation,[6] rate saturation,[7–9] or both.[10] The drawback of these approaches is that flight envelope protection systems prevent the possibility of achieving the actual performance limits of the aircraft: Control systems designed by using such techniques can be overly conservative because of the safety margin necessary to avoid the unsafe region from being reached during highly aggressive maneuvering. The problem is further complicated by the instability of the unaugmented dynamics of modern aircraft because the stability augmentation system (SAS) that artificially provides the necessary stability margin cannot avoid departures from controlled flight in presence of control saturation. These considerations are at the basis of an increasing interest for antiwindup control schemes, in which the control system is capable of coping with maneuvering segments during which the effectors reach their limits in terms of position and/or rate saturation.

The expression integrator windup (and, hence, antiwindup) originated from the belief that instabilities sparked by the presence of saturation in otherwise stable linear closed-loop systems were essentially due to integrators in proportional–integral–derivative (PID) controllers: The state variable of the integral element increases dramatically, that is, winds up, when the actuators reach their position limits and the errors remain large for sizable time intervals, causing degradation of performance, insurgence of wide-amplitude limit cycles, or even instabilities. Subsequently, it was recognized that any kind of controller (even static ones) can be subject to similar performance degradation and/or stability loss effects due to saturation, so that the reference to integrators was dropped, though the expression windup remained standard.

Note that "control in presence of saturation" and "antiwindup compensation" are not synonyms. In techniques devoted to control in the presence of saturation, the design objective is to come up with a compensator K capable of achieving closed-loop stability and, possibly, some kind of performance, for example, asymptotic tracking of feasible references, despite the presence of input saturation (Fig. 1a). (See, for example, Refs. 11–13 and references therein, and see also Refs. 6–10 for applications to flight control problems.) There is no restriction on the form of K and the only a priori knowledge used in the design of K is the open-loop plant dynamics and the saturation level. Conversely, in the antiwindup framework, an unconstrained controller K_M, designed and possibly optimized for the plant to be controlled in the absence of input saturation is assigned as part of the problem data. (See, for example, Refs. 14–18 and references therein.) In this latter case, the design objective is to come up with a compensator K_{aw} connected as an add-on to the

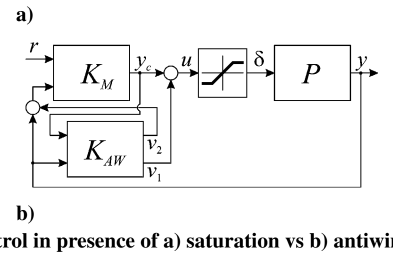

Fig. 1 Control in presence of a) saturation vs b) antiwindup compensation.

saturated closed-loop system (Fig. 1b) that is capable of recovering from the adverse effects of input saturation, meanwhile preserving the preassigned closed-loop response, that is, $v_1 = v_2 = 0$, as long as saturation does not activate.

In the last decade, antiwindup techniques consisting of ad hoc methods, developed for PID controllers and lacking formal problem statements and stability proofs, have been replaced by modern antiwindup strategies, with formal stability proofs and that are applicable to controllers of arbitrary structure. Following the approach proposed in Ref. 18, the requirements for an antiwindup system are as follows: 1) The response induced by the preassigned unconstrained compensator K_M is not modified until saturation occurs. 2) Instabilities due to input saturation during transients are avoided, but the output of the unconstrained closed loop due to any command that is feasible at steady state is asymptotically tracked. To accomplish the foregoing task in the case of open-loop exponentially unstable systems,[19] the antiwindup controller must guarantee the following additional property: 3) The state of the plant never leaves the null controllable region, that is, the set of states that can be driven to zero with bounded inputs, which is a strict subset of the state space in the open-loop exponentially unstable case.

The case of the F-16 can be considered as an example of ad hoc antiwindup compensation on aircraft. An elementary antiwindup scheme was implemented on the F-16 to limit the risk of integrator windup in the proportional–integral element of the longitudinal stability and control augmentation system. In such a scheme, the output of the integrator is bounded to a value equal to the stabilator deflection limit, and the input is reduced by an amount proportional to the saturation violation of the stabilator command.[5] This scheme strictly satisfies only requirement 1, providing a heuristic, although reasonable, approach for the fulfillment of requirements 2 and 3. On the other hand, formal results guaranteeing all three properties with application to flight control systems have been recently proposed. In Ref. 20, an H_∞ optimization approach based on linear matrix inequalities is proposed for the analysis and design of static antiwindup compensators, with application to the linearized longitudinal dynamics of the F-8 fighter aircraft, considering only actuator magnitude saturation. In Ref. 21, the approach proposed in Ref. 19 was extended to account for rate (in addition to magnitude) saturation in the synthesis of an antiwindup compensator, and the special case of manual flight control of an exponentially unstable aircraft [the tailless advanced fighter aircraft (TAFA)] was considered in Ref. 22, wherein a very effective antiwindup compensator is designed. The present paper aims at complementing Refs. 20 and 22 by studying the issue of robustness with respect to variations of trim condition and the associated variations of aircraft stability derivatives, that is, variations of plant parameters.

The accuracy of the model is of paramount importance for the results in Refs. 20 and 22, inasmuch as when an uncertain model is dealt with, the performance of the antiwindup compensator is no longer guaranteed; more precisely, though performance deterioration can be shown to be arbitrarily small for sufficiently small uncertainties,[18] essentially by using suitable small gain arguments, the presence of large uncertainties combined with the antiwindup requirement 1 can spark instabilities even in cases when the preassigned controller was robust to the same uncertainties.[23] Such an observation motivated the introduction of a weakened antiwindup problem, in which requirement 1 is relaxed to allow a robustification of the antiwindup closed loop with respect to larger uncertainties.[23] Clearly, the issue of robustness is crucial when dealing with aircraft capable of reaching high angles of attack, where nonlinear and/or unsteady aerodynamic effects significantly affect the behavior of the vehicle. As an example, Fig. 2 shows the real and imaginary parts of the eigenvalues for the reduced short-period longitudinal model of the TAFA, in a set of trim conditions that ranges between 170 and 600 kn of equivalent airspeed, that is, a dynamic pressure between 100 and 1200 psf.

Such significant changes of the aircraft dynamic behavior usually require the use of gain scheduling, for adapting the SAS to changes in stability derivatives, but this may not be possible when robust multi-input/multi-output controllers are synthesized using modern

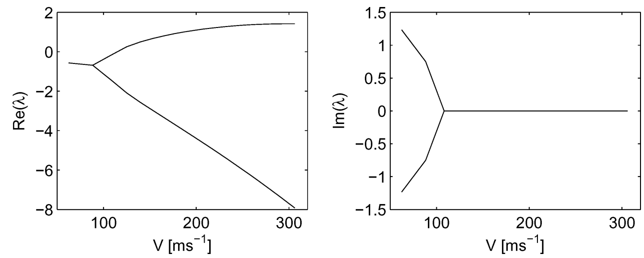

Fig. 2 Plant eigenvalues as function of equivalent airspeed.

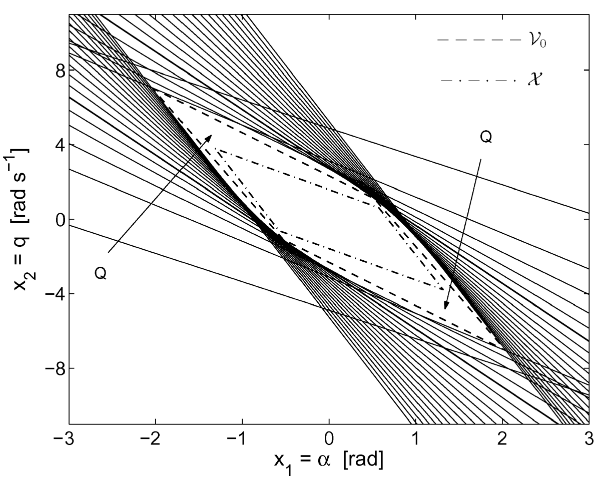

Fig. 3 Safe region $\mathcal{V}$, approximation $\mathcal{V}_0$, and domain of attraction $\mathcal{X}$ for 100 < Q < 1200 psf.

techniques, such as μ synthesis. In such a case, the structure of the controller can change for different operating points, and bumpless transfer techniques are needed for managing the switching of control authority between different controllers. In this circumstance, it is useful to extend as much as possible the operating envelope of each controller, to keep the number of switching thresholds as low as possible. The capability of a robust antiwindup compensator to work far from the nominal operating point, possibly with performance close to the performance of the preassigned compensator, becomes, thus, a relevant property.

With consideration for simplicity of only magnitude saturation at a given trim condition with dynamic pressure Q, the null controllable region for the simplified TAFA model in Ref. 22 is an infinite stripe $\mathcal{V}_Q$ in the state space, with a limited width in the direction of the eigenvector of the unstable mode (Fig. 3). Variations in Q can change the shape of the region $\mathcal{V}_Q$, leaving the state of the aircraft outside of it and dooming any recovery attempt. It is easy to see that such a problem remains even if not only the preassigned controller, but also the antiwindup compensator, is gain scheduled; in fact, a necessary condition for recoverability independently of Q is that the state never leaves the much smaller and, of course, not infinite "safe region," $\mathcal{V} = \bigcap_{Q\in(100,1200)} \mathcal{V}_Q$, represented in Fig. 3.

In this paper, the approach proposed in Ref. 23 for the robustification of antiwindup compensators is extended to unstable plants and is applied for the first time in the framework of manual flight control considering only magnitude saturation. The proposed compensator solves a suitably weakened antiwindup problem for the reduced short-period longitudinal model of the TAFA, successfully coping with plant dynamics variations and keeping the state inside the safe region $\mathcal{V}$, without any scheduling in the considered range of variation of dynamic pressure (100–1200 psf). Compared with previous literature on manual flight control for unstable aircraft with saturating actuators, the main contribution of this paper consists in proposing an approach that yields the reduction of conservativeness, that is, more maneuverable aircraft, achieved by antiwindup strategies coupled with enhanced robustness to aircraft parameter variations.

After the introduction of some notations in the next section, the solution of the weakened antiwindup problem for robust control of a linear unstable plant will be derived. Then, the TAFA model will be described and the design of the antiwindup compensator presented, together with some simulations that demonstrate the viability and effectiveness of the technique. The last section provides some conclusions.

Some Preliminaries

For a given closed convex set $\mathcal{C}\subset\mathbb{R}^p$ and a vector $\boldsymbol{u}\in\mathbb{R}^p$, let $\mathrm{dist}_{\mathcal{C}}(\boldsymbol{u}) := \inf_{w\in\mathcal{C}}(|\boldsymbol{u}-\boldsymbol{w}|)$, where $|\cdot|$ represents the Euclidean norm; $\mathrm{int}(\mathcal{C})$ is the interior of $\mathcal{C}$, and the function $\Gamma_{\mathcal{C}}$ is a projection function on $\mathcal{C}$, that is, $\Gamma_{\mathcal{C}}(\boldsymbol{v})\in\mathcal{C}$, $\forall \boldsymbol{v}\in\mathbb{R}^p$, and $\Gamma_{\mathcal{C}}(\boldsymbol{v})=\boldsymbol{v}$ if $\boldsymbol{v}\in\mathcal{C}$. Note that such a definition actually leaves some degrees of freedom; to fix ideas, the specific choice $\Gamma_{\mathcal{C}}(\boldsymbol{v}) = \arg\inf_{w\in\mathcal{C}}(|\boldsymbol{u}-\boldsymbol{w}|)$ can be considered. Given two vectors $\boldsymbol{x}$ and $\boldsymbol{y}$, their stacking $[\boldsymbol{x}'\ \ \boldsymbol{y}']'$ will be denoted $(\boldsymbol{x},\boldsymbol{y})$. The $\mathcal{L}_2$ norm of a signal $\boldsymbol{w}(\cdot)$ is defined as

$$\|\boldsymbol{w}\|_2 := \sqrt{\int_0^\infty |\boldsymbol{w}(t)|^2\,\mathrm{d}t}$$

and $\boldsymbol{w}\in\mathcal{L}_2$ if $\|\boldsymbol{w}\|_2<\infty$. A system W with input $(\boldsymbol{u},\boldsymbol{v})$ and output $(\boldsymbol{y},\boldsymbol{z})$ is said to have finite incremental ($\mathcal{L}_2$ induced) gain $\gamma_{y,u}^{(W)}\in\mathbb{R}_{\geq 0}$ from $\boldsymbol{u}$ to $\boldsymbol{y}$ if, for any initial condition and any pair of inputs $\boldsymbol{u}_1(\cdot)$ and $\boldsymbol{u}_2(\cdot)$, it holds that

$$\|y(\cdot;\boldsymbol{u}_1,\boldsymbol{v}) - y(\cdot;\boldsymbol{u}_2,\boldsymbol{v})\|_2 \leq \gamma_{y,u}^{(W)}\|\boldsymbol{u}_1-\boldsymbol{u}_2\|_2$$

where $\boldsymbol{y}(t;\boldsymbol{u},\boldsymbol{v})$ is the output response at time t to the given initial condition (not shown in the notation for simplicity) and the inputs are $\boldsymbol{u}(\cdot)$ and $\boldsymbol{v}(\cdot)$. If W is linear time invariant (LTI) and asymptotically stable with transfer function $W(s)$, its incremental gain is equal to $\|W(s)\|_\infty := \sup_{\omega\in\mathbb{R}}\bar{\sigma}(W(j\omega))$, where $\bar{\sigma}(\cdot)$ is the maximum singular value of the argument. Any trivial system (whose output is identically null for any input) is indicated by $\mathbf{0}$ and has zero incremental gain.

To ease the comparison with existing results,[18,19,22] the first part of the paper deals with uncertain systems P_Ψ obtained by the connection of a model P and a perturbation Ψ, where P is given by

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{\delta} + \boldsymbol{w} \tag{1a}$$

$$\boldsymbol{z} = \boldsymbol{C}_1\boldsymbol{x} + \boldsymbol{D}_1\boldsymbol{\delta} \tag{1b}$$

$$\boldsymbol{y} = \boldsymbol{C}_2\boldsymbol{x} + \boldsymbol{D}_2\boldsymbol{\delta} \tag{1c}$$

where $\boldsymbol{y}$ is the measured output, $\boldsymbol{z}$ is a performance output, and $\boldsymbol{\delta}=\mathrm{sat}(\boldsymbol{u})$ is a saturated version of the control input $\boldsymbol{u}$, where $\boldsymbol{\delta}=\boldsymbol{u}$ in the linear model. Here, $\boldsymbol{w}$ is equal to the output $\boldsymbol{y}_\Psi$ of a perturbation Ψ belonging to a family $\mathcal{S}$ of incrementally stable systems,

$$\dot{\boldsymbol{\psi}} = \boldsymbol{f}_\Psi(\boldsymbol{\psi},\boldsymbol{x},\boldsymbol{\delta}) \tag{2a}$$

$$\boldsymbol{y}_\Psi = \boldsymbol{h}_\Psi(\boldsymbol{\psi},\boldsymbol{x},\boldsymbol{\delta}) \tag{2b}$$

where different elements of $\mathcal{S}$ can have different state spaces. Obviously, $\mathbf{0}\in\mathcal{S}$ is assumed, and so P_0 is the nominal model.

For $\rho\in\mathbb{R}_{>0}$, define the family $\mathcal{S}_\rho := \{\Psi\in\mathcal{S} : \gamma^{(\Psi)}_{y_\Psi,u_\Psi} < \rho\}$, that is, the subset of $\mathcal{S}$ containing only uncertainties with incremental gain less than ρ from $\boldsymbol{u}_\Psi = (\boldsymbol{x},\boldsymbol{\delta})$ to $\boldsymbol{y}_\Psi$. Let Σ_Ψ denote a closed-loop system containing P_Ψ as a subsystem, then affected by the same uncertainty $\Psi\in\mathcal{S}$ affecting P_Ψ. A property, for example, $\mathcal{L}_2$ stability, possibly possessed by Σ_Ψ is 1) nominal if possessed by Σ_Ψ when $\Psi=\mathbf{0}$, 2) robust in the small (with respect to $\mathcal{S}$) if possessed by Σ_Ψ for all $\Psi\in\mathcal{S}_\rho$ for some $\rho\in\mathbb{R}_{>0}$, and 3) robust in the large (with respect to $\mathcal{S}$) if possessed by Σ_Ψ for all $\Psi\in\mathcal{S}$. Note that a property is robust in the small if it holds for a subset of $\mathcal{S}$ containing only Ψ having a sufficiently small incremental gain.

In the antiwindup problem, a controller K_M designed for the unsaturated system P_Ψ is a priori given,

$$\dot{\boldsymbol{x}}_c = \boldsymbol{A}_c\boldsymbol{x}_c + \boldsymbol{B}_c\boldsymbol{u}_c + \boldsymbol{E}_c\boldsymbol{r} \tag{3a}$$

$$\boldsymbol{y}_c = \boldsymbol{C}_c\boldsymbol{x}_c + \boldsymbol{D}_c\boldsymbol{u}_c + \boldsymbol{F}_c\boldsymbol{r} \tag{3b}$$

with $\boldsymbol{r}$ being a reference signal. The goal is to design an antiwindup compensator K_{aw},

$$\dot{\boldsymbol{x}}_{\mathrm{aw}} = \boldsymbol{f}_{\mathrm{aw}}(\boldsymbol{x}_{\mathrm{aw}}, \boldsymbol{y}, \boldsymbol{y}_c) \tag{4a}$$

$$\boldsymbol{v}_1 = \boldsymbol{h}_{\mathrm{aw},1}(\boldsymbol{x}_{\mathrm{aw}}, \boldsymbol{y}, \boldsymbol{y}_c) \tag{4b}$$

$$\boldsymbol{v}_2 = \boldsymbol{h}_{\mathrm{aw},2}(\boldsymbol{x}_{\mathrm{aw}}, \boldsymbol{y}, \boldsymbol{y}_c) \tag{4c}$$

that generates suitable signals $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$ that can be seen as corrections to the outputs of K_M and P_Ψ, respectively, thus ensuring good properties for the overall saturated closed-loop system. Because it will be useful to have shorthand notations to refer to different interconnections of P_Ψ, K_M, and K_{aw}, define the following closed-loop systems: 1) the unsaturated closed-loop system Σ_U formed by Eqs. (1–3) when $\boldsymbol{\delta} = \boldsymbol{u} = \boldsymbol{y}_c$ and $\boldsymbol{u}_c = \boldsymbol{y}$; 2) the saturated closed-loop system Σ_S formed by Eqs. (1–3) when $\boldsymbol{\delta} = \mathrm{sat}(\boldsymbol{u})$, $\boldsymbol{u} = \boldsymbol{y}_c$, and $\boldsymbol{u}_c = \boldsymbol{y}$; 3) the unsaturated antiwindup closed-loop system $\Sigma_{U\,\mathrm{aw}}$ formed by Eqs. (1–4) when $\boldsymbol{\delta} = \boldsymbol{u} = \boldsymbol{y}_c + \boldsymbol{v}_1$ and $\boldsymbol{u}_c = \boldsymbol{y} + \boldsymbol{v}_2$; and 4) the (saturated) antiwindup closed-loop system $\Sigma_{S\,\mathrm{aw}}$ formed by Eqs. (1–4) when $\boldsymbol{\delta} = \mathrm{sat}(\boldsymbol{u})$, $\boldsymbol{u} = \boldsymbol{y}_c + \boldsymbol{v}_1$, and $\boldsymbol{u}_c = \boldsymbol{y} + \boldsymbol{v}_2$. Different superscripts denote a signal related to a system, for example, the state $\boldsymbol{x}$ of P, in a particular closed-loop system: An overbar denotes the signal in Σ_U, for example, $\bar{\boldsymbol{x}}$ for the state of P as a subsystem of Σ_U; a tilde denotes the signal in $\Sigma_{U\,\mathrm{aw}}$, for example, $\tilde{\boldsymbol{x}}$ for the state of P as a subsystem of $\Sigma_{U\,\mathrm{aw}}$; and no superscript denotes the signal in $\Sigma_{S\,\mathrm{aw}}$, for example, $\boldsymbol{x}$ for the state of P as a subsystem of $\Sigma_{S\,\mathrm{aw}}$.

Robust Antiwindup Compensator

As outlined in the Introduction, a key step in the construction of a robust-in-the-large antiwindup compensator is to consider a weakened form of antiwindup, as defined in Ref. 23. Its extension to the case of unstable systems, such as an unstable aircraft model, is similar to that proposed in Ref. 19. To make the paper self-contained and to help the reader in appreciating the complementarity between the results in Ref. 22 and those presented here, the $\mathcal{L}_2$ antiwindup and the weakened antiwindup problems are recalled and compared to clarify the relaxation involved in the latter; then the weakened problem is extended to deal with unstable controlled systems; and, finally, the simplifications for the considered aircraft design are highlighted.

The global $\mathcal{L}_2$ antiwindup problem and its solution were defined in Ref. 18 for the case of an open-loop asymptotically stable plant, together with some local results for the unstable case. The statements are here rephrased for notational coherence. In the sequel, $\mathcal{U}$ denotes an arbitrary compact convex subset of the interior of the region on which the saturation is an identity.

Definition 1: The robust-in-the-small $\mathcal{L}_2$ antiwindup problem for $\mathcal{U} \subset \mathbb{R}^p$ and $\mathcal{S}$ is to find an antiwindup compensator such that the following conditions hold.

1) If $\boldsymbol{x}_{\mathrm{aw}}(0) = 0$ and $\bar{\boldsymbol{u}}(\cdot) \equiv \mathrm{sat}[\bar{\boldsymbol{u}}(\cdot)]$, then $z(\cdot) \equiv \bar{z}(\cdot)$.

2) If $\mathrm{dist}_{\mathcal{U}}[\bar{\boldsymbol{u}}(\cdot)] \in \mathcal{L}_2$, then $(z - \bar{z})(\cdot) \in \mathcal{L}_2$. These conditions hold for all $\Psi \in \mathcal{S}$ with sufficiently small incremental gain. □

The following assumption was introduced in Ref. 18 for the foregoing problem to make sense.

Assumption 1: System Σ_U is well-posed and internally stable $\forall \Psi \in \mathcal{S}$. □

Though Assumption 1 requires Σ_U to be well posed and internally stable robustly in the large in $\mathcal{S}$, it is possible to show that, in general, under requirement 1 in Definition 1 only robust-in-the-small performance and stability can be guaranteed a priori for $\Sigma_{S\,\mathrm{aw}}$. As a consequence, the large parameter variations considered for the proposed application to the TAFA dynamics augmented by a given unconstrained compensator prevent the direct application of the extensions to the unstable case[19,22] of the $\mathcal{L}_2$ antiwindup problem and its solution. Note that such an unfortunate fact is due to a form of instability that can arise in any antiwindup compensation satisfying item 1 in Definition 1 when large uncertainties are allowed[23] (see the discussion on $F \circ \Psi$ after Definition 2); hence, it should not be seen as a drawback of the antiwindup compensation proposed in Refs. 18, 19, and 22, but as a performance/robustness tradeoff that is intrinisic in the antiwindup problem.

The same robustness issues occur in any other antiwindup compensator satisfying the mentioned requirement, independently from the design approach used. Because for the TAFA model and the given preassigned controller the problem in Definition 1 has no robust-in-the-large solution, if no measure of the dynamic pressure is used, the requirements in Definition 1 must be relaxed to have a meaningful problem whose solution results in a robustly in-the-large well-behaved control system $\Sigma_{S\,\mathrm{aw}}$. This can be done as in the weakened global $\mathcal{L}_2$ antiwindup problem introduced in Ref. 23, which is extended in this paper to provide an antiwindup compensator for the unstable TAFA model.

To start with, the weakened antiwindup problem is recalled in Definition 2. Two mild assumptions are needed for the problem to make sense. The mildness of Assumption 3 can be appreciated recalling that, with bounded inputs, global asymptotic controllability robust with respect to arbitrary small errors in $\boldsymbol{A}$ requires that $\boldsymbol{A}$ is Hurwitz. As for Assumption 2, note that it is even milder than Assumption 1; moreover, when compared to Assumption 1, it is clear that Assumption 2 is compatible with larger classes of unconstrained controllers K_M, hence allowing for choices of K_M achieving higher performance, at least in the nominal case $\Psi = \boldsymbol{0}$.

Assumption 2: System Σ_U is well-posed and internally stable for $\Psi = \boldsymbol{0}$. □

Assumption 3: $\exists \gamma_{x,\delta}^{(P_\Psi)}, \gamma_{y_\Psi,\delta}^{(P_\Psi)} \in \mathbb{R}_{\geq 0}$ such that, for system P_Ψ,

$$\|\boldsymbol{x}[\cdot;(\boldsymbol{x}_0,\boldsymbol{\psi}_0),\boldsymbol{\delta}_1] - \boldsymbol{x}[\cdot;(\boldsymbol{x}_0,\boldsymbol{\psi}_0),\boldsymbol{\delta}_2]\|_2 \leq \gamma_{x,\delta}^{(P_\Psi)}\|\boldsymbol{\delta}_1 - \boldsymbol{\delta}_2\|_2 \qquad \forall \Psi \in \mathcal{S}$$

$$\|\boldsymbol{y}_\Psi[\cdot;(\boldsymbol{x}_0,\boldsymbol{\psi}_0),\boldsymbol{\delta}_1] - \boldsymbol{y}_\Psi[\cdot;(\boldsymbol{x}_0,\boldsymbol{\psi}_0),\boldsymbol{\delta}_2]\|_2 \leq \gamma_{y_\Psi,\delta}^{(P_\Psi)}\|\boldsymbol{\delta}_1 - \boldsymbol{\delta}_2\|_2 \qquad \forall \Psi \in \mathcal{S} \quad □$$

Definition 2: The weakened global $\mathcal{L}_2$ antiwindup problem for $\mathcal{U}$ with domain of robustness $\mathcal{S}$ is to find an antiwindup compensator such that the following conditions hold.

1) For $\Psi = \boldsymbol{0}$, $\forall \bar{\boldsymbol{u}}(\cdot)$, such that $\Gamma_{\mathcal{U}}[\bar{\boldsymbol{u}}(\cdot)] = \bar{\boldsymbol{u}}(\cdot)$, $\exists \boldsymbol{x}_{\mathrm{aw}}^0 : \boldsymbol{x}_{\mathrm{aw}}(0) = \boldsymbol{x}_{\mathrm{aw}}^0 \Rightarrow z(\cdot) \equiv \bar{z}(\cdot)$.

2) $\Sigma_{U\,\mathrm{aw}}$ is well-posed and internally stable, $\forall \Psi \in \mathcal{S}$.

3) If $\mathrm{dist}_{\mathcal{U}}[\tilde{\boldsymbol{u}}(\cdot)] \in \mathcal{L}_2$, then $(z - \tilde{z})(\cdot) \in \mathcal{L}_2$. □

To understand why the problem in Definition 2 is a weakened version of the problem in Definition 1, note that the key difference between the two problems is a tradeoff between antiwindup performance and robustness. In fact, item 1 of Definition 1 requires the small signal response of $\Sigma_{S\,\mathrm{aw}}$ to match the response of Σ_U for all possible perturbations, whereas item 1 of Definition 2 is only the corresponding nominal requirement. The robust/nominal dichotomy is reflected in the fact that Assumption 2 is only a nominal version of Assumption 1. Both item 3 of Definition 2 and item 2 of Definition 1 assess the effectiveness of the antiwindup response for large signals by limiting the $\mathcal{L}_2$ difference between the responses of $\Sigma_{S\,\mathrm{aw}}$ and another system that is well behaved robustly with respect to $\mathcal{S}$; however, the comparison system in Definition 2 is $\Sigma_{U\,\mathrm{aw}}$ and not Σ_U as in Definition 1. Note that in Definition 2, Σ_U need not even be stable for all $\Psi \in \mathcal{S}$; instead, a robust well-posednature and stability of the comparison system $\Sigma_{U\,\mathrm{aw}}$ is required in item 2 of Definition 2. Because robust stability of Σ_U is not required, the design of K_M can be completely focused on performance, whereas robustness with respect to both saturations and uncertainties will be in charge of the antiwindup compensator, as will be explained now.

The proofs of the theorems that solve the foregoing problems[23,18] exploit a coordinate transformation by which $\Sigma_{S\,\mathrm{aw}}$ is rewritten as an equivalent system (shown in Fig. 4) in which the unmeasured signal $\boldsymbol{y}_\Psi$ appears filtered by an LTI n-input, n-output system F with state $\boldsymbol{x}_F \in \mathbb{R}^{n_F}$. The transfer matrix $\boldsymbol{F}(s)$ of F is an identity in the case of $\mathcal{L}_2$ antiwindup, whereas $\boldsymbol{F}(s)$ is an arbitrary rational proper transfer

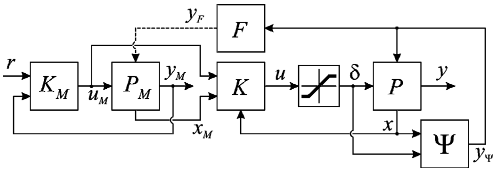

Fig. 4 Equivalent block diagram of robust antiwindup compensator.

matrix in the case of weakened antiwindup. The output $\boldsymbol{y}_F$ of F enters a copy P_M of P, with $\boldsymbol{w}=\boldsymbol{y}_F$ and $\boldsymbol{\delta}=\boldsymbol{u}_M$. The robustness properties of the proposed solutions are shown exploiting the fact that for $F\circ\Psi=\boldsymbol{0}$, the equivalent system has suitable stability properties that are preserved on the grounds of a small gain argument: If $\boldsymbol{F}(s)=\boldsymbol{I}$, Assumption 1 holds, and Ψ is sufficiently small, as in $\mathcal{L}_2$ antiwindup. If $\Psi\in\mathcal{S}$, Assumption 3 holds, and K and $\boldsymbol{F}(s)$ are suitably small, as in weakened antiwindup. In general, the small gain condition allows for enough freedom to guarantee additional properties for $\Sigma_{S\,\mathrm{aw}}$ by a suitable choice of $\boldsymbol{F}(s)$.[23] In particular, though $\boldsymbol{F}=\boldsymbol{0}$ would guarantee $F\circ\Psi=\boldsymbol{0}$, easing the achievement of robust stability, when specific constant [or sinusoidal $\boldsymbol{r}(t)=\sin(\omega t+\phi)$] signals must be tracked, it is desirable to have $\boldsymbol{F}(0)=\boldsymbol{I}$ [$\boldsymbol{F}(j\omega)=\boldsymbol{I}$] to ensure that asymptotic reference tracking and disturbance rejection properties guaranteed by K_M in Σ_U are preserved in $\Sigma_{S\,\mathrm{aw}}$.

For the TAFA model, the instabilities witnessed for large $\Psi\in\mathcal{S}$ when $\mathcal{L}_2$ antiwindup is employed are because $F\circ\Psi=\Psi$ makes $\Sigma_{U\,\mathrm{aw}}$ unstable for such Ψ, in spite of the fact that, by Assumption 1, Σ_U is asymptotically stable for every admissible perturbation $\Psi\in\mathcal{S}$. However, because for the TAFA manual control the steady state (cruise) value of the reference (the desired pitch rate) can be considered zero after some transients (maneuvers), the choice $F=\boldsymbol{0}$ is feasible from the tracking point of view and can be used to solve a weakened antiwindup problem. In this case, the system P_M reduces to the nominal model P_0. Moreover, though in Refs. 18 and 23 the scheme in Fig. 4 is simply an analysis tool used in the proofs obviously, when $F=\boldsymbol{0}$, the scheme can be used for implementation, too. (Direct implementation of the dashed arrow is impossible because $\boldsymbol{y}_\Psi$ cannot be measured.) Note that, though in a somewhat different framework not involving antiwindup, similar model following control systems have been proposed for a long time,[24,25] also with applications in the field of flight control.[26]

There are two main issues to be dealt with in the required extension: First, the antiwindup compensator must guarantee that the state of the controlled system never leaves the safe region $\mathcal{V}$. Second, Assumption 3, which was quite mild in the global case because asymptotic stability of the perturbed plant was almost necessary, must be replaced by some other suitable assumption because even the nominal system is now unstable and then its $\mathcal{L}_2$ gain is not finite. Both issues are taken into account on the basis of the following assumption, basically requiring the knowledge of a robust stabilizer with suitable properties. In the statement, $[\boldsymbol{x}_M(\cdot),\boldsymbol{u}_M(\cdot)]$ is any couple of measurable time functions with $\boldsymbol{x}_M(t)\in\mathbb{R}^n$, $\boldsymbol{u}_M(t)\in\mathbb{R}^p$, $\forall t\geq 0$. A convex safe region $\mathcal{V}\subset\mathbb{R}^n$ is given, and it is desirable to have $\boldsymbol{x}(t)\in\mathcal{V}$, $\forall t\geq 0$.

Assumption 4: There exists a function $\boldsymbol{k}(\cdot,\cdot,\cdot):\mathbb{R}^n\times\mathbb{R}^n\times\mathbb{R}^p\to\mathbb{R}^p$, and two closed convex sets $\mathcal{X}\subset\mathbb{R}^n$ and $\mathcal{X}_0\subset\mathbb{R}^n$ with $\mathcal{X}\subset\mathcal{V}$ and $0\in\mathcal{X}_0\subset\mathrm{int}(\mathcal{X})$, such that 1) $\boldsymbol{x}=\boldsymbol{x}_M$, $(\boldsymbol{x},\boldsymbol{u}_M)\in\mathcal{X}_0\times\mathcal{U}\Rightarrow\boldsymbol{k}(\boldsymbol{x},\boldsymbol{x}_M,\boldsymbol{u}_M)=\boldsymbol{u}_M$; 2) $\boldsymbol{x}(0)\in\mathcal{X}\Rightarrow\boldsymbol{x}\{t;\boldsymbol{x}(0),\boldsymbol{k}[\boldsymbol{x},\boldsymbol{x}_M(t),\boldsymbol{u}_M(t)]\}\in\mathcal{V}$, $\forall t\geq 0$, $\forall\Psi\in\mathcal{S}$, $\forall[\boldsymbol{x}_M(\cdot),\boldsymbol{u}_M(\cdot)]$; and 3) $\lim_{t\to+\infty}[\boldsymbol{x}_M(t),\boldsymbol{u}_M(t)]=0$, $\boldsymbol{x}(0)\in\mathcal{X}\Rightarrow\lim_{t\to+\infty}\boldsymbol{x}\{t;\boldsymbol{x}(0),\boldsymbol{k}[\boldsymbol{x}(t),\boldsymbol{x}_M(t),\boldsymbol{u}_M(t)]\}=0$, $\forall\Psi\in\mathcal{S}$. □

Though the design of $\boldsymbol{k}(\cdot,\cdot,\cdot)$ may look like quite a hard task, in many practical cases the elementary approach outlined in the following section for the TAFA model can be effectively used; for more difficult cases, results based on set invariance and output admissible sets can be exploited, as will be commented on later. Notice that function $\boldsymbol{k}(\cdot,\cdot,\cdot)$ in Assumption 4 corresponds to a static choice for controller K in Fig. 4; though dynamic choices for K can be considered in the proposed framework, static choices for K are usually very satisfactory in applications.

Definition 3: The weakened antiwindup problem for $\mathcal{U}$ with domain of robustness $\mathcal{S}$, domain of attraction $\mathcal{X}$, domain of nominal behavior $\mathcal{X}_0\times\mathcal{U}$, state constraint $\mathcal{V}$, and asymptotically null references, is to find an antiwindup compensator such that, for all $\boldsymbol{x}(0)\in\mathcal{X}$, the following conditions hold.

1) For $\Psi=\boldsymbol{0}$, $\forall[\bar{\boldsymbol{x}}(\cdot),\bar{\boldsymbol{u}}(\cdot)]$ such that $\Gamma_{\mathcal{X}_0}[\bar{\boldsymbol{x}}(\cdot)]=\bar{\boldsymbol{x}}(\cdot)$, $\Gamma_{\mathcal{U}}[\bar{\boldsymbol{u}}(\cdot)]=\bar{\boldsymbol{u}}(\cdot)$, there exists $\boldsymbol{x}_{\mathrm{aw}}^0$: $\boldsymbol{x}_{\mathrm{aw}}(0)=\boldsymbol{x}_{\mathrm{aw}}^0\Rightarrow\boldsymbol{z}(\cdot)\equiv\bar{\boldsymbol{z}}(\cdot)$.

2) $\Gamma_{\mathcal{V}}[\boldsymbol{x}(\cdot)]=\boldsymbol{x}(\cdot)$, $\forall\Psi\in\mathcal{S}$.

3) The $\lim_{t\to+\infty}\boldsymbol{r}(t)=0\Rightarrow\lim_{t\to+\infty}\boldsymbol{x}(t)=0$, $\forall\psi\in\mathcal{S}$. □

In Definition 3, item 1 is the classical requirement of matching the response induced by the unconstrained controller, restricted to the nominal case ($\Psi=\boldsymbol{0}$), as in weakened antiwindup, and to the region $\mathcal{X}_0\times\mathcal{U}$ because following the nominal response outside such region could lead the state outside $\mathcal{V}$. The satisfaction of item 2 guarantees that the safe region $\mathcal{V}$ is never left. Finally, item 3 is a weakened replacement of item 3 (and item 2, in some sense) of Definition 2. In fact, if the whole state is considered as the performance output, that is, $\boldsymbol{z}=\boldsymbol{x}$, assuming that 1) $\Sigma_{U\,\mathrm{aw}}$ is well-posed and asymptotically stable for every admissible perturbation $\Psi\in\mathcal{S}$ and that 2) asymptotically null references bear out that $\tilde{\boldsymbol{z}}$ is asymptotically zero, then item 3 of Definition 2 can be replaced by the requirement that $\boldsymbol{x}$ converges to zero. When it is taken into account that $\boldsymbol{x}$ is a bounded signal, this requirement is something less than asking that $\boldsymbol{x}\in\mathcal{L}_2$, and there is no need to require explicitly a well-posed nature and asymptotic stability of $\Sigma_{U\,\mathrm{aw}}$. For this reason, item 2 of Definition 2 is dropped.

The following theorem provides a solution to the problem in Definition 3.

Theorem: Under Assumption 2 and Assumption 4, the following antiwindup compensation having state $\boldsymbol{x}_{\mathrm{aw}}\in\mathbb{R}^n$ is

$$\dot{\boldsymbol{x}}_{\mathrm{aw}}=\boldsymbol{A}\boldsymbol{x}_{\mathrm{aw}}+\boldsymbol{B}\boldsymbol{y}_c \tag{5a}$$

$$\boldsymbol{v}_1=-\boldsymbol{y}_c+\boldsymbol{k}(\boldsymbol{x},\boldsymbol{x}_{\mathrm{aw}},\boldsymbol{y}_c) \tag{5b}$$

$$\boldsymbol{v}_2=-\boldsymbol{y}+\boldsymbol{C}_2\boldsymbol{x}_{\mathrm{aw}}+\boldsymbol{D}_2\boldsymbol{y}_c \tag{5c}$$

[with $\boldsymbol{x}_{\mathrm{aw}}^0=\boldsymbol{x}(0)$ in item 1 of Definition 3] solving the problem in Definition 3. □

Proof: With the just described antiwindup compensation, the overall closed loop is exactly the system in Fig. 4, where $(\boldsymbol{x}_M,\boldsymbol{u}_M)$ is $(\boldsymbol{x}_{\mathrm{aw}},\boldsymbol{y}_c)$ and K implements the control law $\boldsymbol{k}(\boldsymbol{x},\boldsymbol{x}_M,\boldsymbol{u}_M)$ with $F=\boldsymbol{0}$. Items 1–3 of Definition 3 are then straightforward consequences of Assumption 4. □

Note that the choice $F=\boldsymbol{0}$ used in the antiwindup compensator in the preceding theorem possesses the very attractive property that the achievement of robust-in-the-large stability is completely independent from K_M, leaving as many degrees of freedom as possible in the design of $\boldsymbol{k}(\cdot,\cdot,\cdot)$. On the other hand, in weakened antiwindup compensators with $F\neq\boldsymbol{0}$, suitable compatibility requirements between F and $\boldsymbol{k}$ must be imposed, possibly leading to limitations on $\boldsymbol{k}$, for example, limiting the speed of recovery after saturation achieved by $\boldsymbol{k}$.

Results

To apply the theorem to the robust antiwindup synthesis for the reduced short-period longitudinal model of the TAFA, in this section first the linearized model used in Ref. 22 is briefly recalled and the family $\mathcal{S}$ of relevant perturbations is characterized. Then a procedure that can be used to determine $\mathcal{V}$, $\mathcal{X}$, $\mathcal{X}_0$, $\boldsymbol{k}(\cdot,\cdot,\cdot)$ is outlined, and the resulting sets and functions are explicitly given. Finally, numerical simulations are performed and commented on.

The linearized model of interest is described, for dynamic pressure $Q=0.5\rho V^2$ in the range 100–1200 psf, by[22]

$$\dot{\boldsymbol{x}}=(\boldsymbol{A}+\tilde{Q}\Delta_x)\boldsymbol{x}+(\boldsymbol{B}+\tilde{Q}\Delta_u)\mathrm{sat}_M(u) \tag{6}$$

$$\boldsymbol{y}=\boldsymbol{z}=\boldsymbol{x} \tag{7}$$

where $\boldsymbol{x}=(\alpha,q)$ is the state coinciding with both the measured and the performance output, $\tilde{Q}$ is the difference between the actual value of the dynamic pressure $Q \in (100, 1200)$ and the nominal pressure $Q_0 = 450$ psf, and the actuator saturation limit is $M = 0.35$ rad. The nominal ($\tilde{Q} = 0$) dynamics is

$$\boldsymbol{A} = \begin{bmatrix} Z_\alpha^0 & 1 \\ M_\alpha^0 & M_q^0 \end{bmatrix} = \begin{bmatrix} -0.9 & 1.0 \\ 5.9375 & -2.1 \end{bmatrix}, \qquad \boldsymbol{B} = \begin{bmatrix} 0 \\ M_\delta^0 \end{bmatrix} = \begin{bmatrix} 0 \\ 8 \end{bmatrix}$$

The family $\mathcal{S}$ of uncertainties Ψ is $\boldsymbol{y}_\Psi = \tilde{Q}\Delta \boldsymbol{u}_\Psi$, with $\boldsymbol{u}_\Psi = [x, \mathrm{sat}_M(u)]$, $\tilde{Q} \in (-350, 750)$, and $\Delta = [\Delta_x \;\; \Delta_u]$ with

$$\Delta_x = \begin{bmatrix} -\dfrac{1}{750} & 0 \\ \dfrac{3}{160} & -\dfrac{1}{300} \end{bmatrix}, \qquad \Delta_u = \begin{bmatrix} 0 \\ \dfrac{4}{225} \end{bmatrix}$$

As shown in Fig. 2, the plant has two real eigenvalues for $Q > 128$ psf, one of which becomes unstable for $Q > 165.3$ psf. For any fixed value of $Q > 165.3$ psf, the system can be diagonalized in the form

$$\begin{bmatrix} \dot{x}_s \\ \dot{x}_u \end{bmatrix} = \begin{bmatrix} \lambda_s & 0 \\ 0 & \lambda_u \end{bmatrix} \begin{bmatrix} x_s \\ x_u \end{bmatrix} + \begin{bmatrix} b_s \\ b_u \end{bmatrix} \mathrm{sat}_M(u)$$

so that the null controllability region in the (x_s, x_u) variables is the strip $\mathcal{V}_Q = \{(x_s, x_u) \in \mathbb{R}^2 : |x_u| < M|b_u|/|\lambda_u|\}$. In the (α, q) variables, such a region will appear transformed by the diagonalizing change of coordinates, taking the shape of the strip between a couple of parallel lines in $\mathbb{R}^2$, such as that between the bold lines in Fig. 3, for the nominal case $Q = Q_0$.

When Q varies, both λ_u and its eigenvector change, determining the convex safe region $\mathcal{V} = \bigcap_{Q \in (100,1200)} \mathcal{V}_Q$ (the central white area in Fig. 3) such that if $\boldsymbol{x}(t) \notin \mathcal{V}$ for some $t \geq 0$, then the state is outside the null controllability region for at least one value of Q and then is unrecoverable, at least as long as Q is constant at such value. This motivates the need to have $\boldsymbol{x}(t) \in \mathcal{V}$, $\forall t \geq 0$. To construct our antiwindup compensator, we 1) look for a gain matrix L such that $\boldsymbol{A} + \tilde{Q}\Delta_x + (\boldsymbol{B} + \tilde{Q}\Delta_u)L$ is Hurwitz for all admissible values of $\tilde{Q}$; 2) approximate $\mathcal{V}$ by a set $\mathcal{V}_0 \subset \mathcal{V}$ (the dashed parallelogram in Fig. 3); 3) find a set $\mathcal{X} \subset \mathcal{V}_0$ (the dash–dotted parallelogram in Fig. 3) such that for $\dot{\boldsymbol{x}} = (\boldsymbol{A} + \tilde{Q}\Delta_x)\boldsymbol{x} + (\boldsymbol{B} + \tilde{Q}\Delta_u)\mathrm{sat}_M(\boldsymbol{Lx})$, all trajectories starting on $\partial\mathcal{X}$ (the boundary of $\mathcal{X}$) converge to the origin without exiting $\mathcal{V}_0$, for all admissible values of $\tilde{Q}$; and 4) blend the foregoing ingredients to determine $\boldsymbol{k}(\cdot, \cdot, \cdot)$.

When the value of $\boldsymbol{L} = [-1.03 \;\; -0.36]$ corresponding to $u = -(\lambda_u + \epsilon)x_u$ with $\epsilon = 0.52$ for the value of $Q = 412$ psf is used as a first guess it is easy to check that such an $\boldsymbol{L}$ is a suitable choice for step 1. This value of Q was chosen minimizing $M|b_u|/|\lambda_u|$ inside the interval $Q \in (165, 1200)$ where there is an unstable eigenvalue, that is, considering the operating condition where the input has the lowest control authority. As for step 2, a good internal approximation $\mathcal{V}_0$ of $\mathcal{V}$ can be trivially found. As for step 3, a simple MATLAB® routine based on the numerical integration of a system of ordinary differential equations (ODE) was used to determine $\mathcal{X}$ as shown in Fig. 3. In particular, because $n = 2$ and the considered sets $\mathcal{V}_0$ and (candidate) $\mathcal{X}$ are parallelograms, to check if a candidate $\mathcal{X}$ is a suitable choice it is adequate to investigate the behavior of solutions of the ODE starting on $\partial\mathcal{X}$, and such a test can be easily performed. Should a candidate $\mathcal{X}$ fail the test, the preceding analysis can also be used to find which couple of vertices of $\mathcal{X}$ can be moved to find a better candidate, in such a way that convergence to a suitable set $\mathcal{X}$ is ensured. We remark that, though powerful tools based on set invariance and output admissible sets[27,28] can be considered for deriving larger sets $\mathcal{V}$ and $\mathcal{X}$, their use in the present paper was avoided to show that a much simpler yet effective approach can be employed, at least for the considered TAFA model, providing also an α-limiting feature that is particularly relevant for flight control applications.

Letting $\gamma \in (0, 1)$, $\mathcal{X}_0 := \{\boldsymbol{x} : \gamma^{-1}\boldsymbol{x} \in \mathcal{X}\}$, a function $\boldsymbol{k}(\boldsymbol{x}, \boldsymbol{x}_M, \boldsymbol{u}_M)$ satisfying Assumption 4 can be easily given in the form

$$\boldsymbol{k}(\boldsymbol{x}, \boldsymbol{x}_M, \boldsymbol{u}_M) = \boldsymbol{L}[\boldsymbol{x} - \beta(\boldsymbol{x})\Gamma_{\mathcal{X}_0}(\boldsymbol{x}_M)] + \beta(\boldsymbol{x})\Gamma_{\mathcal{U}_1}(\boldsymbol{u}_M)$$

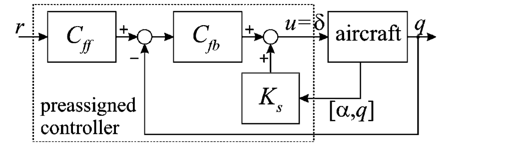

Fig. 5 Unconstrained controller structure.

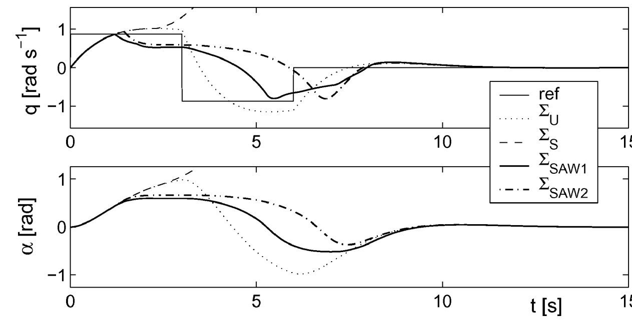

Fig. 6 Response to large command ($r = 50$ deg · s^{-1}), nominal case ($Q = 450$ psf).

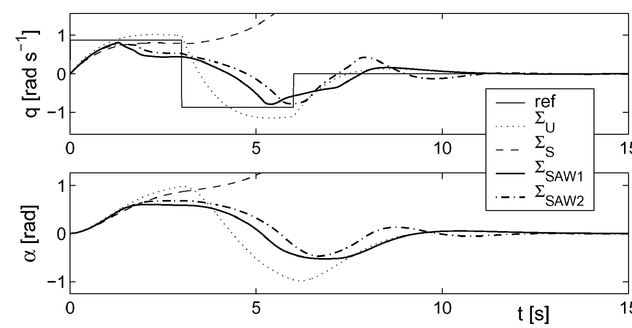

Fig. 7 Response to large command ($r = 50$ deg · s^{-1}), low-speed case ($Q = 350$ psf).

where $\beta(\boldsymbol{x})$ is any continuous function such that $\beta(\boldsymbol{x}) = 1$ if $\boldsymbol{x} \in \mathcal{X}_0$, $\beta(\boldsymbol{x}) = 0$ if $\boldsymbol{x} \notin \mathcal{X}$, and the projections are $\Gamma_{\mathcal{X}_0}(\boldsymbol{x}_M) := (\max\{1, \boldsymbol{F}_1\boldsymbol{x}_M, \boldsymbol{F}_2\boldsymbol{x}_M, \boldsymbol{F}_3\boldsymbol{x}_M, \boldsymbol{F}_4\boldsymbol{x}_M\})^{-1}x_M$, where $\boldsymbol{F}_i$, $i = 1, \ldots, 4$, are such that $\boldsymbol{x}_M \in \mathcal{X}_0$ iff $\boldsymbol{F}_i\boldsymbol{x}_M \leq 1$, for $i = 1, \ldots, 4$ (Ref. 13) and $\Gamma_{\mathcal{U}_1}(u_M) = \mathrm{sat}_M(u_M)$. The function $\Gamma_{\mathcal{U}_1}(\cdot)$, tuned by trial and error by the choice of a convex compact $\mathcal{U}_1 \supseteq \mathcal{U}$, can be used to reduce some transients induced by excessive values of u_M. A value of $\gamma = 0.95$ was used in all of the reported simulations.

The a priori given, unconstrained controller designed in Ref. 22 to meet a prototypical military specification

$$\frac{q(s)}{q_d(s)} = \frac{1.4s + 1}{s^2 + 1.5s + 1}$$

at trim condition Q_0 is shown in Fig. 5 with

$$\boldsymbol{K}_S = -\left(1/M_\delta^0\right)\left[M_\alpha^0 \quad M_q^0\right], \qquad C_{ff} = (1.4s + 1)/(1.5s + 1)$$

$$C_{fb} = \left(1/M_\delta^0\right)(1.5s + 1)/s$$

Several simulations were performed for different values of the dynamic pressure Q, and the responses of Σ_U, Σ_S, and $\Sigma_{S\,\mathrm{aw}}$ were compared. In Figs. 6–8, the dotted and dashed lines represent the response of Σ_U and Σ_S, respectively. The state and control variables of $\Sigma_{S\,\mathrm{aw}}$ are represented by thick continuous lines when the weakened antiwindup compensator presented in this paper is considered (which will be referred to as $\Sigma_{S\,\mathrm{aw}_1}$ in the following text), whereas dash–dotted lines are used when the $\mathcal{L}_2$ scheme of Ref. 22 is employed ($\Sigma_{S\,\mathrm{aw}_2}$ in the sequel). A thin continuous line for the pitch rate q shows the reference signal, used for the considered closed-loop systems, for different values of the dynamic pressure Q.

When feasible maneuvers for the saturated model Σ_S are dealt with, both compensators recover exactly the response of the nominal plant. When dealing with aggressive maneuvers, with $r = q_{\mathrm{des}}$ as

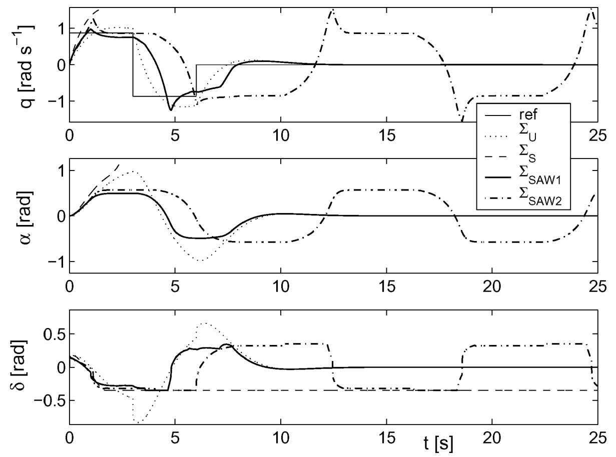

Fig. 8 Response to large command (r = 50 deg · s^{-1}), high-speed case (Q = 900 psf).

high as 50 deg · s^{-1}, as in the reported simulations, the instability of the basic airframe dooms the state of Σ_S as soon as saturation occurs, whereas the two antiwindup strategies provide only slightly different results in the nominal case ($Q = 450$ psf). The differences apparent in Fig. 6 are due to 1) the extension of the safe region, which is wider for $\Sigma_{S\,\mathrm{aw}_2}$, and 2) the signal routing of $\Sigma_{S\,\mathrm{aw}_1}$, where the value of the state variables of the real plant is not fed back to the preassigned compensator K_M. The first property allows $\Sigma_{S\,\mathrm{aw}_2}$ to reach higher peak values in terms of both pitch rate and angle of attack, but the tracking of the nominal unsaturated response is better when $\Sigma_{S\,\mathrm{aw}_1}$ is used, especially as concerns α. From the point of view of the handling qualities, the more conservative, weakened approach apparently limits the aircraft maneuverability in terms of maximum achievable angle of attack (although the difference is marginal), but if the nominal response was carefully tailored to satisfy HQ requirements, its better tracking performance is to be considered a highly desirable characteristic. In particular, the time required to change the direction of the pitching motion becomes sizably smaller, saving, at least qualitatively, the symmetry of the response in terms of angle of attack.

The attractive features of the weakened approach become more evident when off-nominal cases are dealt with. In the low-speed case ($Q = 350$ psf), reported in Fig. 7, the lower control power due to the reduced value of the dynamic pressure results in a degradation of the response of $\Sigma_{S\,\mathrm{aw}_2}$. In particular, the damped oscillations at the end of the maneuver represent an undesirable behavior, whereas $\Sigma_{S\,\mathrm{aw}_1}$ does not seem to suffer from the reduced value of the control derivative M_δ, its tracking capabilities being fairly close to those demonstrated for the nominal case.

A more dramatic improvement is achieved in the high-speed case ($Q = 900$ psf). Figure 8 shows that when M_δ is increased and the instability of the unaugmented plant becomes more severe, $\Sigma_{S\,\mathrm{aw}_2}$ is no longer capable of achieving an equilibrium condition after an aggressive input and undamped oscillations in pitch build up in spite of the fact that the state never leaves the null controllable region. Simulations were run also changing the projection operator, using the reduced safe region of $\Sigma_{S\,\mathrm{aw}_1}$ with $\Sigma_{S\,\mathrm{aw}_2}$, without any significant improvement in the response. Thus, the approach proposed in Ref. 22 is not able to cope with the dramatic variation of system parameters. Conversely, the performance of $\Sigma_{S\,\mathrm{aw}_1}$ are absolutely adequate for the considered task, in all of the considered conditions, without any change in the gains of the controller or the parameters of the antiwindup compensator.

In Fig. 9 it is shown how the antiwindup compensator of $\Sigma_{S\,\mathrm{aw}_1}$ successfully keeps the state inside the prescribed subset $\mathcal{X}$ of the safe region, in the last, more critical case. From the engineering point of view, an interesting byproduct of the proposed approach evident in Fig. 9 is that an inherent capability of angle of attack limiting is achieved. When α becomes large, the maximum achievable value of the pitch rate gets smaller, until for $\alpha > \alpha_{\max}$ the pitch rate must become negative, thus forcing a reduction of the angle of attack.

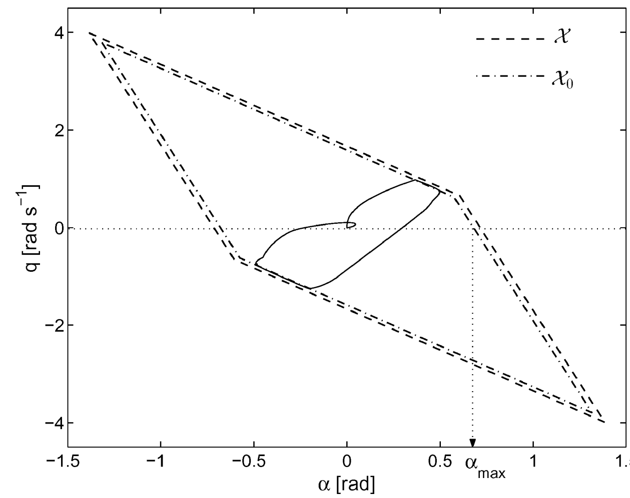

Fig. 9 Sets $\mathcal{X}$ and $\mathcal{X}_0$ and state trajectory.

This means that it is possible to obtain a stall prevention system and/or a load-factor limiter by simply tailoring the bounds of $\mathcal{X}$.

Finally, it can be observed by simulations that it is sufficient to size properly the subset $\mathcal{X}$ to maintain the performance of the closed-loop system with an antiwindup compensator also in the presence of rate saturated actuators. This latter feature was not systematically investigated, thus remaining out of the rigorous formalization presented in the preceding paragraphs. The extension of the proposed approach to the case of rate-saturated actuators, as well as the feasibility of the proposed design for higher-order models containing either a more complete dynamic description and/or additional dynamic uncertainties, such as a complete longitudinal model, actuator dynamics, or flexural modes, represent the natural progression of this work.

Conclusions

The weakened antiwindup approach discussed in this paper represents an extension of the $\mathcal{L}_2$ antiwindup technique, where some performance in terms of tracking accuracy is traded for achieving compensator robustness with respect to large plant parameter variations, such as those experienced by an aircraft because of changes of trim condition. This robustification technique is applied for the first time to an unstable plant, that is, the reduced short-period model of a highly maneuverable modern fighter aircraft.

The proposed approach provides robust stability augmentation in the whole set of trim conditions for the TAFA model, in the subsonic speed range for which aerodynamic derivatives were available, keeping the state of the aircraft inside the safe region in which recovery is possible in spite of control saturation and large parameter variations. An effective, although heuristic, approach was envisioned for determining such a region. As is usual with robust controllers, the enhanced robustness possessed by the proposed compensator comes at the price of some conservativeness; however, proper shaping of the safe region also yields a useful, intrinsic angle-of-attack limiting capability that is a good property when a flight control application is dealt with.

Acknowledgments

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References

[1] Smith, J. W., "Analysis of a longitudinal Pilot-Induced Oscillation Experienced on the Approach and Landing Test of the Space Shuttle," NASA TM 81366, Dec. 1981.

[2] Anderson, S. B., "Historical Survey of Flight-Testing Accidents," NASA Tech Briefs, Rept. ARC-13330, Sept. 1995.

[3] Duda, H., "Prediction of Pilot-in-the–Loop Oscillations Due to Rate Saturation," *Journal of Guidance, Control, and Dynamics*, Vol. 20, No. 3, 1997, pp. 581–587.

[4] Duda, H., Duus, G., Hovmark, G., and Forssell, L., "New Flight Simulator Experiments on Pilot-Involved Oscillations Due to Rate Saturation,"

Journal of Guidance, Control, and Dynamics, Vol. 23, No. 2, 2000, pp. 312–318.

[5]Droste, C. S., and Walker, J. E., *A Case Study on the F-16 Fly-By-Wire Flight Control System*, AIAA Professional Studies Series, AIAA, New York, 1980, pp. 1–20.

[6]Pachter, M., Chandler, P. R., and Smith, L., "Maneuvering Flight Control," *Journal of Guidance, Control, and Dynamics*, Vol. 21, No. 3, 1998, pp. 361–367.

[7]Snell, S. A., and Hess, R. A., "Robust, Decoupled, Flight Control Design with Rate-Saturating Actuators," *Journal of Guidance, Control, and Dynamics*, Vol. 21, No. 3, 1998, pp. 361–367.

[8]Miller, R. B., and Pachter, M., "Maneuvering Flight Control with Actuator Constraints," *Journal of Guidance, Control, and Dynamics*, Vol. 20, No. 4, 1997, pp. 729–734.

[9]Lin, Z., Pachter, M., Banda, S., and Shamash, Y., "State and Output Feedback Design for Robust Tracking of Linear Systems with Rate Limited Actuators," *Optimal Control Applications and Methods*, Vol. 23, No. 1, 2002, pp. 21–43.

[10]Snell, S. A., and Hess, R. A., "Robust, Decoupled, Flight Control Design with Rate-Saturating Actuators," *Journal of Guidance, Control, and Dynamics*, Vol. 20, No. 1, 1997, pp. 90–96.

[11]Lin, Z., "Semiglobal Stabilization of Linear Systems with Position and Rate Limited Actuators," *Systems and Control Letters*, Vol. 30, No. 1, 1997, pp. 1–11.

[12]Gilbert, E. G., Kolmanovsky, I., and Tan, K. T., "Discrete Time Reference Governors and the Nonlinear Control of Systems with State and Control Constraints," *International Journal of Robust and Nonlinear Control*, Vol. 5, No. 5, 1995, pp. 487–504.

[13]Blanchini, F., and Miani, S., "Any Domain of Attraction for a Linear Constrained System Is a Tracking Domain of Attraction," *SIAM Journal on Control and Optimization*, Vol. 38, No. 3, 2000, pp. 971–994.

[14]Hanus, R.,"Antiwindup and Bumpless Transfer: A Survey," *Proceedings of the 12th World Congress on Scientific Computation*, Vol. 2, IMACS, Paris, July 1988, pp. 59–65.

[15]Kothare, M. V., Campo, P. J., Morari, M., and Nett, C. N., "A Unified Framework for the Study of Antiwindup Designs," *Automatica*, Vol. 30, No. 12, 1994, pp. 1869–1883.

[16]Miyamoto, A., and Vinnicombe, G., "Robust Control of Plants with Saturation Nonlinearity Based on Coprime Factor Representation," *36th IEEE Conference on Decision and Control*, Vol. 3, Dec. 1996, pp. 2838–2840.

[17]Edwards, C., and Postlethwaite, I., "An Anti-Windup Scheme with Closed-Loop Stability Considerations," *Automatica*, Vol. 35, No. 4, 1999, pp. 761–765.

[18]Teel, A. R., and Kapoor, N., "The $\mathcal{L}_2$ Anti-Windup Problem: Its Definition and Solution," Proceedings of the 4th European Control Conf., Paper 494, July 1997.

[19]Teel, A. R., "Anti-Windup for Exponentially Unstable Linear Systems," *International Journal of Robust and Nonlinear Control*, Vol. 9, No. 10, 1999, pp. 701–716.

[20]Cao, Y.-Y., Lin, Z., and Ward, D. G., "H_∞ Antiwindup Design for Linear Systems Subject to Input Saturation," *Journal of Guidance, Control, and Dynamics*, Vol. 25, No. 3, 2002, pp. 455–463.

[21]Barbu, C., Reginatto, R., Teel, A. R., and Zaccarian, L., "Anti-Windup for Exponentially Unstable Linear Systems with Inputs Limited in Magnitude and Rate," *IEEE American Control Conference*, Vol. 2, June 2000, pp. 1230–1234.

[22]Barbu, C., Reginatto, R., Teel, A. R., and Zaccarian, L., "Anti-Windup Design for Manual Flight Control," *IEEE American Control Conference*, Vol. 2, June 1999, pp. 3186–3190.

[23]Galeani, S., "Two Degrees of Freedom Control for Systems with Constraints," Ph.D. Dissertation, Dip. di Informatica Sistemi e Produzione, Univ. di Roma "Tor Vergata," Rome, June 2002.

[24]Moore, J. B., Xia, L., and Glover, K., "On Improving Control-Loop Robustness of Model-Matching Controllers," *Systems and Control Letters*, Vol. 7, No. 2, 1986, pp. 83–87.

[25]Tyler, J., Jr., "The Characteristics of Model-Following Systems as Synthesized by Optimal Control," *IEEE Transactions on Automatic Control*, Vol. 9, No. 4, 1964, pp. 485–498.

[26]Kocurek, J. N., and Durham, W. C., "Dynamic Inversion and Model-Following Flight Control: A Comparison of Performance Robustness," AIAA Paper 97-3777, Aug. 1997.

[27]Blanchini, F., "Set Invariance in Control—A Survey," *Automatica*, Vol. 35, No. 11, 1999, pp. 1747–1767.

[28]Gilbert, E. G., and Tan, K. T., "Linear Systems with State and Control Constraints: The Theory and Application of Maximal Output Admissible Sets," *IEEE Transactions on Automatic Control*, Vol. 36, No. 9, 1991, pp. 1008–1020.